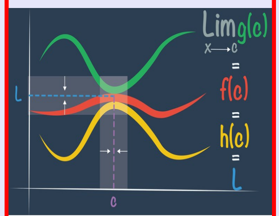
Calculus I

Lecture 45



Feb 19-8:47 AM

Class QZ 16 (open notes)

Find abs. Max & abs. Min. of

$f(x) = 5 + 54x - 2x^3$ on $[0,4]$.

$\boxed{f(x) \text{ is a polynomial}} \Rightarrow \boxed{\text{Cont. \& Diff. } (-\infty,\infty)}$

$f'(x) = 54 - 6x^2$

$f'(x) = 0$

$54 - 6x^2 = 0$

$x^2 = 9$

$\boxed{x=3}$ $\cancel{x = -3}$

$f(0) = 5$ ✓ , $f(4) = 93$ ✓

$f(3) = 113$ ✓

Abs. Max @ $(3, 113)$

Abs. Min @ $(0, 5)$

Abs. max. Value 113

Abs. Min. Value 5

Nov 19-8:25 AM

More on integration

1) $\int f(x)\,dx = F(x) + C$ where $\frac{d}{dx}[F(x)+C] = f(x)$

2) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$. Power rule

3) $\int \cos x\,dx = \sin x + C$

4) $\int \sin x\,dx = -\cos x + C$

5) $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

6) $\int c f(x)\,dx = c\int f(x)\,dx$

7) $\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$ Definite Integral

Nov 19-7:27 AM

Find $\int (x\sqrt{x} - \sqrt[5]{x})\,dx$

$= \int \left[x \cdot x^{1/2} - x^{1/5} \right] dx$

$= \int \left[x^{\frac{3}{2}} - x^{1/5} \right] dx$

$= \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} - \frac{x^{\frac{1}{5}+1}}{\frac{1}{5}+1} + C$

$= \frac{x^{5/2}}{5/2} - \frac{x^{6/5}}{6/5} + C = \frac{2}{5}x^{5/2} - \frac{5}{6}x^{6/5} + C$

$= \frac{2}{5}\sqrt{x^5} - \frac{5}{6}\sqrt[5]{x^6} + C$

$= \boxed{\frac{2}{5}x^2\sqrt{x} - \frac{5}{6}x\sqrt[5]{x} + C}$

Nov 20-7:31 AM

Evaluate $\int_1^4 \left[\sqrt{x} + \frac{1}{\sqrt{x}}\right]dx$

$$= \int_1^4 \left[x^{1/2} + x^{-1/2}\right]dx$$

$$= \left(\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right)\Bigg|_1^4 = \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{1/2}}{1/2}\right)\Bigg|_1^4$$

$$= \left(\frac{2}{3}x\sqrt{x} + 2\sqrt{x}\right)\Bigg|_1^4 = \left(\frac{2}{3}\cdot 4\sqrt{4} + 2\sqrt{4}\right) - \left(\frac{2}{3}\cdot 1\cdot\sqrt{1} + 2\sqrt{1}\right)$$

$$= \boxed{\frac{16}{3}} + 4 \boxed{-\frac{2}{3}} - 2 = \frac{14}{3} + 2 = \frac{14}{3} + \frac{6}{3} = \boxed{\frac{20}{3}}$$

Nov 20-7:36 AM

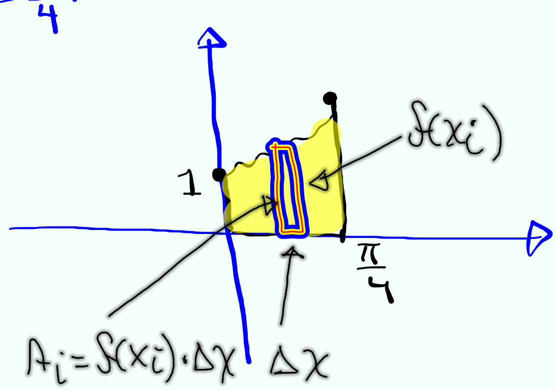
Find the area below $f(x) = \sec^2 x$, above x-axis from $x = 0$ to $x = \frac{\pi}{4}$.

$f(x) = \sec^2 x > 0$

Cont. on $\left[0, \frac{\pi}{4}\right]$

$f(0) = \sec^2 0 = 1$

$f\left(\frac{\pi}{4}\right) = \sec^2 \frac{\pi}{4} = 2$

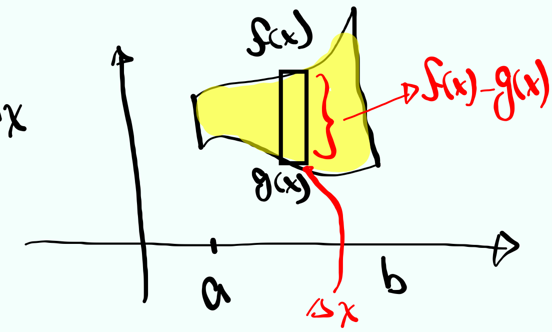


$$A = \int_0^{\frac{\pi}{4}} \sec^2 x \, dx = \tan x \Big|_0^{\pi/4} = \cancelto{1}{\tan\frac{\pi}{4}} - \cancelto{0}{\tan 0} = \boxed{1}$$

Nov 20-7:41 AM

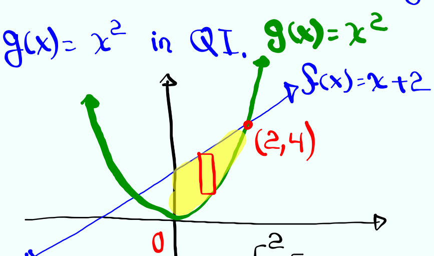
If $f(x) \geq g(x)$ on $[a,b]$, then the area below $f(x)$, above $g(x)$ from $x=a$ to $x=b$ is given by

$$A = \int_a^b [f(x) - g(x)]\,dx$$



Nov 20-7:46 AM

Find the area bounded by $f(x) = x+2$ and $g(x) = x^2$ in QI.



$x^2 = x+2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x=2 \quad x=-1$

$$A = \int_0^2 [\text{Top} - \text{Bottom}]\,dx$$

$$= \int_0^2 (x+2-x^2)\,dx$$

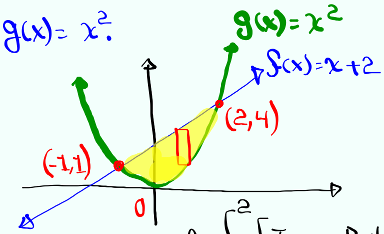
$$= \left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)\Big|_0^2 = \left(\frac{2^2}{2} + 2(2) - \frac{2^3}{3}\right) - \cancel{\left(\frac{0^2}{2} + 2(0) - \frac{0^3}{3}\right)}^{0}$$

$$= 2 + 4 - \frac{8}{3} = 6 - \frac{8}{3}$$

$$= \frac{18}{3} - \frac{8}{3} = \boxed{\frac{10}{3}}$$

Nov 20-7:50 AM

Find the area bounded by $f(x)=x+2$ and $g(x)=x^2$.



$x^2 = x+2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x=2 \quad x=-1$

$$A = \int_{-1}^{2} [\text{Top} - \text{Bottom}]\,dx$$

$$= \int_{-1}^{2} (x+2 - x^2)\,dx$$

$$= \left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)\Bigg|_{-1}^{2}$$

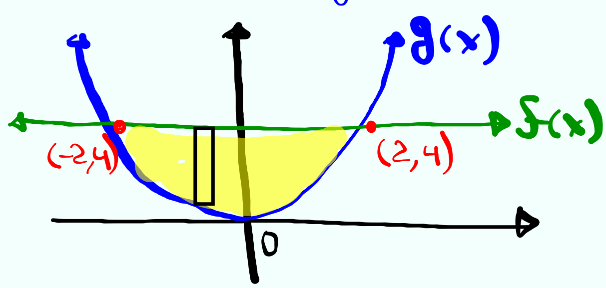
$$= \left[\frac{2^2}{2} + 2(2) - \frac{2^3}{3}\right] - \left[\frac{(-1)^2}{2} + 2(-1) - \frac{(-1)^3}{3}\right]$$

$$= \boxed{\frac{4}{2}} + 4 \boxed{-\frac{8}{3}} \boxed{-\frac{1}{2}} + 2 \boxed{-\frac{1}{3}}$$

$$= \frac{3}{2} + 6 - \frac{9}{3} = \frac{3}{2} + 6 - 3 = \frac{3}{2} + 3 = \boxed{\frac{9}{2}}$$

Nov 20-7:50 AM

Find the area enclosed by $f(x)=4$ & $g(x)=x^2$.



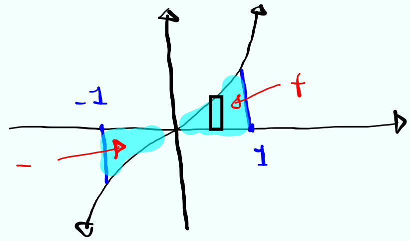
$$A = \int_{-2}^{2} [4 - x^2]\,dx = 2\int_{0}^{2} [4 - x^2]\,dx = 2\left[4x - \frac{x^3}{3}\right]\Bigg|_{0}^{2}$$

$$= 2\left[4(2) - \frac{2^3}{3} - 0\right] = 2\left(8 - \frac{8}{3}\right) = 2 \cdot \frac{16}{3}$$

$$= \boxed{\frac{32}{3}}$$

Nov 20-8:02 AM

Find the area enclosed by $f(x)=x^3$ and x-axis from $x=-1$ to $x=1$.



$[0,1]$ Top x^3
Bottom x-axis $y=0$

$[-1,0]$ Top x-axis $y=0$
Bottom x^3

$$A=\int_{-1}^{1}(x^3-0)\,dx=\int_{-1}^{1}x^3\,dx=\frac{x^4}{4}\Big|_{-1}^{1}=\ldots=\cancel{0}$$

$$A=2\int_0^1(x^3-0)\,dx=2\cdot\frac{x^4}{4}\Big|_0^1=\frac{1}{2}(1^4)-0$$

$$=\boxed{\frac{1}{2}}\checkmark$$

Nov 20-8:19 AM

$$\int(2x-1)\,dx=x^2-x+C$$

$$\int(2x-1)^2\,dx=\int(4x^2-4x+1)\,dx=\frac{4x^3}{3}-2x^2+x+C$$

$$\int(2x-1)^3\,dx=\int\left[(2x)^3-3(2x)^2+3(2x)-1\right]dx$$

$$=\int(8x^3-12x^2+6x-1)\,dx$$

$$=\frac{8x^4}{4}-\frac{12x^3}{3}+\frac{6x^2}{2}-x+C$$

$$=2x^4-4x^3+3x^2-x+C$$

Nov 20-8:26 AM

$\int (\underbrace{2x-1}) ^3 dx$ by <u>Subs. method</u>

Let $u = 2x - 1$

$du = 2dx \rightarrow \frac{du}{2} = dx$

$$\int (2x-1)^3 dx = \int u^3 \cdot \frac{du}{2} = \frac{1}{2} \int u^3 du$$

$$= \frac{1}{2} \cdot \frac{u^4}{4} + C$$

$$= \boxed{\frac{1}{8}(2x-1)^4 + C}$$

Nov 20-8:30 AM